

conserved quantities general relativity

Understanding Conserved Quantities in General Relativity

conserved quantities general relativity represent some of the most profound and persistent concepts in our understanding of gravity and the universe. While Einstein's theory of general relativity fundamentally altered our perception of spacetime, replacing Newtonian notions of absolute space and time with a dynamic, interwoven fabric, the core principles of conservation laws remain remarkably relevant. These conserved quantities, such as energy, momentum, and angular momentum, are not merely mathematical conveniences but are deeply tied to the symmetries of spacetime itself, as elucidated by Noether's theorem. Exploring conserved quantities in the context of general relativity is crucial for comprehending phenomena ranging from the orbits of planets to the dynamics of black holes and the evolution of the cosmos. This article will delve into the intricate relationship between symmetries, conserved quantities, and the geometric structure of spacetime in general relativity, offering a detailed look at their implications and the challenges in their definition.

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Introduction to Conserved Quantities in Physics

Conserved quantities are fundamental pillars of physics, embodying principles that have stood the test of time across numerous theories. At their heart, these quantities are physical properties of a system that remain constant over time, regardless of the interactions or transformations the system undergoes. Think of it like a closed box: whatever happens inside, the total amount of stuff that started in the box will still be there at the end. This concept is incredibly powerful because it allows us to predict and constrain the behavior of complex systems without needing to meticulously track every single interaction. From the simplest mechanical pendulum to the intricate dance of subatomic particles, the laws of conservation – of energy, momentum, angular momentum, and

charge – provide an essential framework for understanding the universe.

In classical mechanics, these conservation laws are directly observable and relatively straightforward to define. For instance, the total momentum of a system of colliding billiard balls remains unchanged before and after the collision, assuming no external forces are acting upon them. Similarly, the total energy, whether kinetic or potential, of an isolated system will always stay the same. These principles simplify analysis immensely, allowing physicists to bypass detailed calculations of every individual force. However, as we venture into the realm of Einstein's general relativity, the landscape becomes significantly more nuanced. The very fabric of spacetime is no longer a passive backdrop but an active participant in gravitational interactions, leading to a fascinating re-evaluation of these seemingly immutable conserved quantities.

Symmetries and Noether's Theorem

The deep connection between symmetries and conserved quantities is one of the most elegant discoveries in theoretical physics, formalized by Emmy Noether. Her groundbreaking theorem, published in 1918, states that for every continuous symmetry of a physical system, there corresponds a conserved quantity. This is a profound statement: it means that if a system's laws of physics remain unchanged under a certain transformation (a symmetry), then there's a specific quantity associated with that transformation that will be conserved. For example, the laws of physics in most systems are the same today as they were yesterday, and will be tomorrow. This time translation symmetry leads to the conservation of energy. Likewise, if you move from one point in space to another and the laws of physics remain the same, that spatial translation symmetry gives rise to the conservation of linear momentum. Similarly, rotational symmetry leads to the conservation of angular momentum.

Noether's theorem is not just a theoretical curiosity; it's a guiding principle that has shaped our understanding of fundamental physics. It tells us that conservation laws aren't arbitrary rules but are intrinsically linked to the fundamental structure of reality. The elegance lies in its universality. It applies not only to classical mechanics but also to quantum field theory and, as we will explore, general relativity. By identifying the symmetries of a given physical situation, we can directly deduce the associated conserved quantities, significantly simplifying the problem and offering deep insights into the underlying physics. This theorem provides a powerful lens through which to view the universe, revealing hidden order and constancy within apparent change.

Conserved Quantities in Newtonian Gravity

In the context of Newtonian gravity, conserved quantities are relatively straightforward and are deeply intertwined with the assumed properties of absolute space and time. Here, space is a fixed, Euclidean stage, and time flows uniformly and independently for all observers. Because of these assumptions, Newtonian gravity exhibits clear symmetries. The laws of motion and gravitational interaction are the same no matter where you are in

space (spatial translation symmetry), no matter which direction you are facing (rotational symmetry), and no matter when you are observing the system (time translation symmetry).

These symmetries directly translate into the familiar conservation laws. The conservation of linear momentum arises from the fact that the gravitational force between two objects depends only on their relative positions, not their absolute location in space. If you shift the entire system of planets in the solar system to a different location, the forces between them would remain identical, and thus their total momentum would be conserved. Similarly, the conservation of angular momentum stems from the fact that gravity acts radially and the solar system is assumed to be spherically symmetric. Rotational symmetries mean that if you spin the entire solar system around its center, the forces and motions would be preserved, leading to constant total angular momentum. Finally, the conservation of energy, encompassing both kinetic and potential gravitational energy, is a direct consequence of the time-translation symmetry – the laws governing planetary orbits don't change over time in Newtonian physics. These conserved quantities provide the bedrock for understanding celestial mechanics for centuries.

General Relativity and the Challenge of Spacetime

General relativity, Einstein's revolutionary theory of gravity, fundamentally alters our understanding of spacetime. It posits that gravity is not a force acting across space, but rather a manifestation of the curvature of spacetime itself, caused by the presence of mass and energy. This geometric interpretation introduces a significant challenge to the straightforward application of conserved quantities as understood in Newtonian physics. In general relativity, spacetime is dynamic; it can bend, warp, and even ripple. This means that spacetime itself is no longer a static, absolute background but is inextricably linked to the matter and energy within it.

The core difficulty arises from the fact that general relativity is a generally covariant theory. This means its equations are formulated in a way that is independent of the chosen coordinate system. While this covariance is essential for the theory's success, it makes it challenging to define global conserved quantities like total energy or momentum for an isolated system. In Newtonian physics, we can easily define an inertial frame of reference where we can measure absolute energy and momentum. However, in general relativity, there isn't a universally agreed-upon, preferred inertial frame. Spacetime can be curved, and what one observer measures as energy, another observer in a different, perhaps accelerating, frame might measure something else entirely, or the concept of "total energy of the universe" becomes ill-defined. This doesn't mean conservation laws are broken, but rather that their formulation requires a more sophisticated approach, deeply tied to the specific symmetries of the spacetime geometry.

Killing Vector Fields: The Key to Symmetries

To address the challenges of identifying symmetries in the curved and dynamic spacetime of general relativity, physicists turn to the concept of Killing vector fields. A Killing vector field is a vector field on a spacetime manifold whose flow preserves the metric of the spacetime. In simpler terms, if you "drag" spacetime along the direction of a Killing vector field, the distances and angles within that spacetime remain unchanged. These vector fields are the mathematical embodiment of continuous symmetries in a curved spacetime. Each distinct Killing vector field corresponds to a unique symmetry, and by Noether's theorem, each symmetry implies a conserved quantity.

For instance, a time-like Killing vector field represents a time translation symmetry. If a spacetime possesses a time-like Killing vector field, it means that the physics of that spacetime are invariant under translation along the direction of that vector. This symmetry is crucial for defining a conserved quantity related to energy. Similarly, space-like Killing vector fields correspond to spatial translation symmetries (leading to momentum conservation) and rotational symmetries (leading to angular momentum conservation). The number and nature of Killing vector fields present in a specific spacetime geometry dictate the conserved quantities associated with that spacetime. So, rather than having universal conservation laws applicable everywhere in any gravitational configuration, general relativity links conserved quantities to the specific geometric symmetries present in the local or global structure of spacetime itself. This makes the search for conserved quantities a geometric problem.

Energy and Momentum in General Relativity

Defining and conserving energy and momentum in general relativity is a notoriously complex topic, largely due to the absence of a universally agreed-upon metric for these quantities in all situations. As discussed, the lack of preferred inertial frames and the dynamic nature of spacetime make a straightforward Newtonian definition problematic. However, for spacetimes that possess certain symmetries, we can still recover meaningful conserved quantities. In spacetimes with a time-like Killing vector field (a symmetry under time translation), one can define an "energy" that is conserved along the integral curves of this vector field. This is often referred to as the "energy associated with the symmetry."

For spacetimes that are asymptotically flat – meaning they resemble flat spacetime far away from any gravitational sources, like our universe on large scales – we can define conserved quantities related to energy and momentum at spatial infinity. These are often referred to as the ADM mass (for Arnowitt, Deser, and Misner) and ADM momentum. The ADM mass, in this context, represents the total energy content of the gravitational field plus any matter and radiation, measured at infinity. Similarly, the ADM momentum represents the total momentum of the system at infinity. These are extremely useful concepts for describing isolated systems like the solar system or binary black hole mergers. However, for spacetimes that are not asymptotically flat, or that lack specific symmetries, defining and conserving global energy and momentum becomes significantly more challenging, and these quantities may not be well-defined or conserved in the traditional sense.

Angular Momentum in General Relativity

Angular momentum, like energy and linear momentum, presents a nuanced picture in general relativity. In spacetimes exhibiting rotational symmetry, characterized by the presence of a space-like Killing vector field that corresponds to rotations, the component of angular momentum along the axis of symmetry is conserved. This is a direct application of Noether's theorem: the symmetry of rotation implies the conservation of angular momentum around that axis.

For spacetimes that are asymptotically flat, analogous to the ADM formulation for energy and momentum, one can define conserved quantities related to angular momentum at spatial infinity. These are often referred to as the Komar integrals, which are generalizations of the ADM quantities. These conserved angular momentum components describe the total angular momentum of an isolated system far from any gravitational influences. For systems like rotating stars or galaxies, where rotational symmetry is a key feature, these conserved quantities are essential for understanding their dynamics. However, the concept of total angular momentum becomes more intricate in more complex spacetimes, particularly those without clear rotational symmetries or in the presence of exotic gravitational phenomena.

Conserved Quantities in Specific Spacetimes

The nature and number of conserved quantities in general relativity are highly dependent on the specific geometric structure of the spacetime. Different solutions to Einstein's field equations describe different gravitational environments, each with its own set of symmetries and, consequently, its own set of conserved quantities.

- **Schwarzschild Spacetime:** This is the spacetime describing a non-rotating, spherically symmetric black hole or a star. It possesses two time-like Killing vector fields (one for time translation and one for rotations around any axis due to spherical symmetry) and two space-like Killing vector fields (corresponding to rotations around any axis). This leads to the conservation of energy and angular momentum for objects moving in this spacetime.
- **Kerr Spacetime:** This describes a rotating, uncharged black hole. It possesses a time-like Killing vector field and a space-like Killing vector field corresponding to rotations about the symmetry axis. This allows for the conservation of energy and axial angular momentum. However, due to the lack of full rotational symmetry, angular momentum about axes perpendicular to the rotation axis is not globally conserved in the same way as in Schwarzschild spacetime.
- **Flat Spacetime (Minkowski Spacetime):** This is the spacetime of special relativity, representing the absence of gravity. It is highly symmetric, possessing 10 independent Killing vector fields (4 for time/space translations, 6 for rotations and boosts). These symmetries lead to the conservation of energy, linear momentum, and angular momentum for all systems within this spacetime.

- **Cosmological Spacetimes (e.g., FLRW):** The Friedmann–Lemaître–Robertson–Walker (FLRW) metric describes a homogeneous and isotropic universe. These spacetimes possess a high degree of symmetry, leading to conserved quantities associated with energy and momentum within this cosmological context. However, the interpretation of these quantities can differ from isolated systems.

Understanding these specific spacetimes and their associated symmetries is key to grasping how conserved quantities manifest in different physical scenarios governed by general relativity.

Challenges and Limitations

Despite the power of Killing vector fields and Noether's theorem, defining and working with conserved quantities in general relativity is not without its challenges and limitations. One of the most significant hurdles is the difficulty in identifying these symmetries in complex, realistic spacetimes. Many astrophysical scenarios, such as the merger of two black holes or the dynamics of a turbulent accretion disk, do not possess simple, global symmetries. In such cases, the existence of Killing vector fields might be limited to local regions, or they might not exist at all, rendering the direct application of Noether's theorem for global conserved quantities impossible.

Furthermore, the concept of "energy" in general relativity is particularly problematic in situations where there are no time-like Killing vectors. In such scenarios, energy is not a well-defined global quantity. This is not to say that energy is not conserved, but rather that its conservation is more subtle and might be tied to more intricate properties of the spacetime or specific definitions that are context-dependent. The energy-momentum pseudo-tensor, for example, is often used to discuss the energy-momentum of the gravitational field itself, but it is not locally defined and depends on the choice of coordinates, which can lead to ambiguity. These complexities highlight that while conserved quantities are fundamental, their precise formulation and interpretation in the curved dynamic arena of general relativity require careful consideration of the underlying spacetime geometry and its symmetries.

The Future of Conserved Quantities in Relativity

The ongoing exploration of conserved quantities in general relativity continues to push the boundaries of our understanding of gravity and the universe. As physicists develop more sophisticated mathematical tools and delve into more extreme astrophysical environments, new avenues for defining and interpreting these fundamental principles are emerging. The quest for a consistent definition of energy in general relativity remains an active area of research, particularly in the context of cosmological spacetimes and situations involving quantum gravity, where the very nature of spacetime might be fundamentally different.

Future research may involve exploring alternative mathematical frameworks that can better capture conserved quantities in non-symmetric spacetimes, perhaps through the development of more robust integral invariants or by leveraging insights from quantum field theory in curved spacetime. The study of gravitational waves, for instance, provides a rich laboratory for testing our understanding of conserved quantities in dynamic gravitational fields. Ultimately, a deeper comprehension of conserved quantities in general relativity is not just an academic exercise; it is essential for unlocking the secrets of black holes, the early universe, and the ultimate fate of the cosmos.

Conclusion

The concept of conserved quantities, while seemingly straightforward in classical physics, takes on a profound and intricate character within the framework of general relativity. The theory's dynamic and geometric interpretation of gravity necessitates a re-evaluation of how we define and understand the conservation of energy, momentum, and angular momentum. The key lies in the symmetries of spacetime, meticulously identified through Killing vector fields, which, by virtue of Noether's theorem, directly correspond to these conserved quantities. While challenges persist, especially in highly complex or non-symmetric spacetimes, the pursuit of understanding these fundamental principles in general relativity continues to illuminate the workings of the universe, from the orbits of planets to the enigmatic nature of black holes. The journey of unraveling conserved quantities in Einstein's theory remains a vital and fascinating frontier in theoretical physics.

Frequently Asked Questions about Conserved Quantities in General Relativity

Q: Are energy and momentum conserved in general relativity?

A: Yes, but their definition and conservation are more nuanced than in Newtonian physics. For spacetimes with sufficient symmetries (like time-translation or spatial-translation symmetries, identified by Killing vector fields), corresponding conserved quantities for energy and momentum can be defined. For asymptotically flat spacetimes, quantities like ADM mass and ADM momentum are conserved at spatial infinity.

Q: What is the role of symmetries in conserved quantities in general relativity?

A: Symmetries are absolutely central. Noether's theorem states that for every continuous symmetry of a physical system, there is a corresponding conserved quantity. In general relativity, these symmetries are represented by Killing vector fields, which preserve the spacetime metric.

Q: What is a Killing vector field?

A: A Killing vector field is a vector field on a spacetime manifold whose flow preserves the spacetime's metric. In simpler terms, it represents a direction in which spacetime is invariant under translation, signifying a continuous symmetry.

Q: How is energy defined in general relativity, especially in curved spacetimes?

A: In spacetimes with a time-like Killing vector field, energy can be defined and is conserved along the integral curves of that vector. In asymptotically flat spacetimes, the ADM mass at infinity serves as a conserved quantity representing the total energy content. However, in generic, non-symmetric spacetimes, a universally applicable definition of total energy is problematic.

Q: Why is it difficult to define total energy for the entire universe in general relativity?

A: This difficulty arises from the general covariance of the theory and the absence of a preferred global inertial frame. Spacetime is dynamic and curved, making it hard to isolate a system and consistently define its total energy in a way that is independent of the observer and coordinate system.

Q: Does angular momentum conservation hold true in general relativity?

A: Yes, for spacetimes with rotational symmetries, characterized by specific space-like Killing vector fields. In such cases, the component of angular momentum along the axis of symmetry is conserved. For asymptotically flat spacetimes, conserved quantities related to angular momentum can be defined at infinity.

Q: Are there any universal conserved quantities that apply to all spacetimes in general relativity?

A: No, not in the same straightforward, universally applicable sense as in Newtonian physics. The specific conserved quantities depend heavily on the symmetries possessed by a particular spacetime geometry. Different solutions to Einstein's field equations will have different sets of conserved quantities.

Q: What are the implications of conserved quantities for phenomena like black hole mergers?

A: For phenomena like black hole mergers in asymptotically flat spacetimes, conserved quantities like ADM mass and ADM linear/angular momentum are extremely important. They allow us to quantify the total energy and momentum radiated away as gravitational

waves and the final state of the merged black hole.

Q: How do conserved quantities relate to the concept of an isolated system in general relativity?

A: The concept of an isolated system is crucial. For a system to have well-defined conserved quantities, it often needs to be considered in a context where it is effectively separated from influences, such as in the asymptotic regions of spacetime where it approaches flatness.

Q: What is the role of the Einstein field equations in understanding conserved quantities?

A: The Einstein field equations describe the relationship between spacetime geometry and matter/energy. Solving these equations yields specific spacetime metrics, and the symmetries of these metrics dictate the existence and nature of conserved quantities through Killing vector fields.

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