

COLLEGE ALGEBRA INTERVAL NOTATION INEQUALITIES

MASTERING COLLEGE ALGEBRA INTERVAL NOTATION AND INEQUALITIES

COLLEGE ALGEBRA INTERVAL NOTATION INEQUALITIES ARE FUNDAMENTAL CONCEPTS THAT UNLOCK A DEEPER UNDERSTANDING OF MATHEMATICAL RELATIONSHIPS AND FUNCTIONS. THEY PROVIDE A CONCISE AND POWERFUL WAY TO REPRESENT SETS OF NUMBERS AND THE RANGES WITHIN WHICH CERTAIN CONDITIONS HOLD TRUE. WHETHER YOU'RE GRAPHING FUNCTIONS, SOLVING COMPLEX EQUATIONS, OR ANALYZING DATA, A FIRM GRASP OF THESE TOOLS IS ESSENTIAL FOR SUCCESS IN COLLEGE ALGEBRA AND BEYOND. THIS ARTICLE WILL GUIDE YOU THROUGH THE INTRICACIES OF INTERVAL NOTATION AND INEQUALITIES, BREAKING DOWN HOW TO INTERPRET, WRITE, AND APPLY THEM WITH CONFIDENCE. WE'LL EXPLORE DIFFERENT TYPES OF INEQUALITIES, FROM SIMPLE LINEAR ONES TO THOSE INVOLVING ABSOLUTE VALUES AND QUADRATIC EXPRESSIONS, AND DEMONSTRATE HOW INTERVAL NOTATION OFFERS A CLEAR AND ELEGANT WAY TO EXPRESS THEIR SOLUTIONS.

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UNDERSTANDING INEQUALITIES

AT ITS CORE, AN INEQUALITY IS A MATHEMATICAL STATEMENT THAT COMPARES TWO EXPRESSIONS USING SYMBOLS LIKE $<$, $>$, $\leq$, or $\geq$. UNLIKE EQUATIONS, WHICH ASSERT EQUALITY, INEQUALITIES DESCRIBE A RELATIONSHIP WHERE ONE SIDE IS GREATER THAN, LESS THAN, GREATER THAN OR EQUAL TO, OR LESS THAN OR EQUAL TO THE OTHER. THINK OF IT AS A COMPARISON OF MAGNITUDES RATHER THAN A DEMAND FOR SAMENESS. FOR INSTANCE, IF WE SAY A VARIABLE ' x ' IS GREATER THAN 5 (WRITTEN AS $x > 5$), WE'RE NOT SAYING ' x ' IS EXACTLY 5; WE'RE DESCRIBING ALL THE NUMBERS THAT ARE LARGER THAN 5.

THESE COMPARISONS ARE CRUCIAL IN MATHEMATICS BECAUSE MANY REAL-WORLD SCENARIOS DON'T INVOLVE EXACT VALUES BUT RATHER RANGES OR LIMITS. FOR EXAMPLE, A SPEED LIMIT ISN'T AN EXACT SPEED YOU MUST DRIVE; IT'S AN UPPER BOUNDARY. SIMILARLY, A MINIMUM AGE REQUIREMENT FOR SOMETHING ISN'T A SINGLE PRECISE AGE; IT'S A LOWER BOUNDARY. INEQUALITIES HELP US MODEL THESE TYPES OF SITUATIONS MATHEMATICALLY, PROVIDING A FRAMEWORK FOR UNDERSTANDING POSSIBILITIES AND CONSTRAINTS. THE SYMBOLS THEMSELVES ARE QUITE INTUITIVE: ' $<$ ' MEANS "LESS THAN," ' $>$ ' MEANS "GREATER THAN," ' $\leq$ ' MEANS "LESS THAN OR EQUAL TO," AND ' $\geq$ ' MEANS "GREATER THAN OR EQUAL TO." THE INCLUSION OF THE "OR EQUAL TO" IN THE LATTER TWO IS A KEY DISTINCTION FROM STRICT INEQUALITIES.

THE IMPORTANCE OF INEQUALITY SYMBOLS

THE INEQUALITY SYMBOLS ARE THE BACKBONE OF ANY INEQUALITY STATEMENT. THEY DICTATE THE NATURE OF THE RELATIONSHIP BETWEEN THE TWO EXPRESSIONS BEING COMPARED. UNDERSTANDING THESE SYMBOLS IS THE FIRST STEP TO CORRECTLY INTERPRETING AND SOLVING ANY INEQUALITY PROBLEM. FOR EXAMPLE, ' $x < 3$ ' MEANS x CAN BE ANY NUMBER SMALLER THAN 3, SUCH AS 2, 1, 0, OR EVEN NEGATIVE NUMBERS. IN CONTRAST, ' $x \geq 0$ ' MEANS x CAN BE 0 OR ANY POSITIVE NUMBER, EFFECTIVELY REPRESENTING ALL NON-NEGATIVE REAL NUMBERS.

WHEN DEALING WITH INEQUALITIES, IT'S ALSO IMPORTANT TO REMEMBER HOW OPERATIONS AFFECT THE INEQUALITY SIGN. MULTIPLYING OR DIVIDING BOTH SIDES OF AN INEQUALITY BY A POSITIVE NUMBER DOES NOT CHANGE THE DIRECTION OF THE

INEQUALITY. HOWEVER, MULTIPLYING OR DIVIDING BY A NEGATIVE NUMBER REVERSES THE DIRECTION OF THE INEQUALITY SIGN. THIS IS A CRITICAL RULE TO KEEP IN MIND WHEN YOU'RE MANIPULATING INEQUALITIES TO ISOLATE A VARIABLE. FORGETTING THIS STEP IS A COMMON PITFALL THAT CAN LEAD TO INCORRECT SOLUTIONS.

TYPES OF INEQUALITIES AND THEIR SOLUTIONS

INEQUALITIES COME IN VARIOUS FORMS, FROM THE STRAIGHTFORWARD LINEAR INEQUALITIES TO MORE COMPLEX QUADRATIC AND ABSOLUTE VALUE INEQUALITIES. EACH TYPE REQUIRES A SLIGHTLY DIFFERENT APPROACH TO FIND ITS SOLUTION SET, WHICH IS THE COLLECTION OF ALL VALUES THAT MAKE THE INEQUALITY TRUE. FOR LINEAR INEQUALITIES, THE SOLUTION IS TYPICALLY AN INFINITE RANGE OF NUMBERS THAT CAN BE EASILY REPRESENTED ON A NUMBER LINE.

ABSOLUTE VALUE INEQUALITIES INTRODUCE A TWIST, AS THEY OFTEN HAVE SOLUTIONS THAT ARE SPLIT INTO TWO DISTINCT INTERVALS OR A SINGLE INTERVAL ENCOMPASSING EVERYTHING BETWEEN TWO POINTS. QUADRATIC INEQUALITIES, WHICH INVOLVE A SQUARED TERM, CAN HAVE SOLUTIONS THAT ARE EITHER TWO SEPARATE INTERVALS OR A SINGLE INTERVAL, AND THEIR SOLUTION PROCESS OFTEN INVOLVES FINDING THE ROOTS OF THE CORRESPONDING QUADRATIC EQUATION. MASTERING THESE DIFFERENT TYPES ENSURES YOU CAN TACKLE A WIDE ARRAY OF PROBLEMS ENCOUNTERED IN COLLEGE ALGEBRA.

LINEAR INEQUALITIES

LINEAR INEQUALITIES ARE THE MOST BASIC FORM, INVOLVING VARIABLES RAISED ONLY TO THE FIRST POWER. THEY RESEMBLE LINEAR EQUATIONS BUT USE INEQUALITY SYMBOLS. FOR INSTANCE, ' $2x + 3 < 7$ ' IS A LINEAR INEQUALITY. SOLVING IT INVOLVES ISOLATING THE VARIABLE ' x ' USING ALGEBRAIC OPERATIONS, MUCH LIKE SOLVING A LINEAR EQUATION, WITH THE CRUCIAL CAVEAT OF REVERSING THE INEQUALITY SIGN IF YOU MULTIPLY OR DIVIDE BY A NEGATIVE NUMBER.

THE SOLUTION TO A LINEAR INEQUALITY IS A SET OF NUMBERS THAT CAN BE REPRESENTED AS AN INTERVAL ON THE NUMBER LINE. FOR ' $2x + 3 < 7$ ', WE SUBTRACT 3 FROM BOTH SIDES TO GET ' $2x < 4$ '. THEN, WE DIVIDE BY 2 TO GET ' $x < 2$ '. THIS MEANS ANY NUMBER LESS THAN 2 SATISFIES THE ORIGINAL INEQUALITY. THIS INFINITE RANGE OF NUMBERS IS WHERE INTERVAL NOTATION TRULY SHINES.

ABSOLUTE VALUE INEQUALITIES

ABSOLUTE VALUE INEQUALITIES INVOLVE THE ABSOLUTE VALUE OF AN EXPRESSION, DENOTED BY VERTICAL BARS (E.G., $|x|$). THE ABSOLUTE VALUE OF A NUMBER IS ITS DISTANCE FROM ZERO, SO IT'S ALWAYS NON-NEGATIVE. AN INEQUALITY LIKE ' $|x| < 5$ ' MEANS THAT THE DISTANCE OF ' x ' FROM ZERO IS LESS THAN 5. THIS TRANSLATES TO ' x ' BEING BETWEEN -5 AND 5, INCLUSIVE OF NEITHER ENDPOINT.

ABSOLUTE VALUE INEQUALITIES OFTEN BREAK DOWN INTO TWO SEPARATE INEQUALITIES. FOR EXAMPLE, ' $|ax + b| < c$ ' IS EQUIVALENT TO ' $-c < ax + b < c$ '. IF THE INEQUALITY IS ' $|ax + b| > c$ ', IT BREAKS INTO TWO DISTINCT INEQUALITIES: ' $ax + b > c$ ' OR ' $ax + b < -c$ '. UNDERSTANDING THESE TRANSFORMATIONS IS KEY TO CORRECTLY SOLVING AND REPRESENTING THE SOLUTIONS OF ABSOLUTE VALUE INEQUALITIES.

QUADRATIC INEQUALITIES

QUADRATIC INEQUALITIES INVOLVE A VARIABLE RAISED TO THE SECOND POWER, SUCH AS ' $x^2 - 4x + 3 > 0$ '. TO SOLVE THESE, WE FIRST FIND THE ROOTS OF THE CORRESPONDING QUADRATIC EQUATION ($x^2 - 4x + 3 = 0$). THESE ROOTS, IN THIS CASE, ARE $x = 1$ AND $x = 3$. THESE ROOTS DIVIDE THE NUMBER LINE INTO THREE INTERVALS: $(-\infty, 1)$, $(1, 3)$, AND $(3, \infty)$.

WE THEN TEST A VALUE FROM EACH INTERVAL IN THE ORIGINAL INEQUALITY TO SEE IF IT HOLDS TRUE. FOR EXAMPLE, TESTING $x = 0$ IN ' $x^2 - 4x + 3 > 0$ ' GIVES $3 > 0$, WHICH IS TRUE. TESTING $x = 2$ GIVES $-1 > 0$, WHICH IS FALSE. TESTING $x = 4$ GIVES $7 > 0$, WHICH IS TRUE. THEREFORE, THE SOLUTION SET FOR ' $x^2 - 4x + 3 > 0$ ' CONSISTS OF THE INTERVALS $(-\infty, 1)$ AND $(3, \infty)$. THIS DEMONSTRATES HOW QUADRATIC INEQUALITIES CAN LEAD TO SOLUTIONS COMPOSED OF MULTIPLE, DISJOINT INTERVALS.

INTRODUCTION TO INTERVAL NOTATION

INTERVAL NOTATION IS A SHORTHAND WAY OF REPRESENTING A RANGE OF NUMBERS ON THE NUMBER LINE. IT'S INCREDIBLY USEFUL FOR EXPRESSING THE SOLUTION SETS OF INEQUALITIES, ESPECIALLY WHEN THOSE SOLUTIONS INVOLVE AN INFINITE NUMBER OF VALUES. INSTEAD OF WRITING OUT "ALL NUMBERS GREATER THAN 3," WE CAN SIMPLY USE INTERVAL NOTATION TO CONVEY THE SAME INFORMATION MORE CONCISELY AND WITH GREATER PRECISION.

THE BEAUTY OF INTERVAL NOTATION LIES IN ITS SIMPLICITY AND CLARITY. IT USES PARENTHESES AND BRACKETS TO INDICATE WHETHER THE ENDPOINTS OF THE INTERVAL ARE INCLUDED OR EXCLUDED. THIS MIGHT SEEM LIKE A SMALL DETAIL, BUT IT'S CRUCIAL FOR ACCURATELY DEFINING THE SET OF NUMBERS. IT'S LIKE DRAWING A LINE ON A NUMBER LINE BUT USING SPECIAL MARKERS TO SHOW EXACTLY WHERE THE LINE BEGINS AND ENDS, AND WHETHER THOSE MARKERS ARE PART OF THE LINE ITSELF.

UNDERSTANDING PARENTHESES AND BRACKETS

IN INTERVAL NOTATION, PARENTHESES $()$ ARE USED TO INDICATE THAT AN ENDPOINT IS NOT INCLUDED IN THE INTERVAL. THIS IS TYPICALLY USED WITH STRICT INEQUALITIES LIKE ' $<$ ' AND ' $>$ '. THINK OF A PARENTHESIS AS AN OPEN CIRCLE ON A NUMBER LINE, SIGNIFYING THAT THE NUMBER ITSELF IS NOT PART OF THE SOLUTION SET.

BRACKETS $[]$ ARE USED TO INDICATE THAT AN ENDPOINT IS INCLUDED IN THE INTERVAL. THIS IS USED WITH INEQUALITIES THAT INCLUDE "OR EQUAL TO," SUCH AS ' $\leq$ ' AND ' $\geq$ '. A BRACKET IS LIKE A CLOSED CIRCLE ON A NUMBER LINE, MEANING THE NUMBER IS A VALID PART OF THE SOLUTION. INFINITY (∞) AND NEGATIVE INFINITY $(-\infty)$ ARE ALWAYS REPRESENTED WITH PARENTHESES BECAUSE THEY ARE NOT REAL NUMBERS AND THUS CANNOT BE INCLUDED AS ENDPOINTS.

COMMON INTERVAL NOTATIONS

THERE ARE A FEW STANDARD WAYS INTERVAL NOTATION IS USED, DEPENDING ON WHETHER THE INTERVAL IS BOUNDED OR UNBOUNDED AND WHETHER THE ENDPOINTS ARE INCLUDED. LET'S BREAK DOWN THE MOST COMMON FORMS YOU'LL ENCOUNTER:

- (a, b) : REPRESENTS ALL NUMBERS BETWEEN ' a ' AND ' b ', EXCLUDING ' a ' AND ' b '. THIS CORRESPONDS TO INEQUALITIES LIKE $a < x < b$.
- $[a, b]$: REPRESENTS ALL NUMBERS BETWEEN ' a ' AND ' b ', INCLUDING ' a ' AND ' b '. THIS CORRESPONDS TO INEQUALITIES LIKE $a \leq x \leq b$.
- $(a, b]$: REPRESENTS ALL NUMBERS BETWEEN ' a ' AND ' b ', EXCLUDING ' a ' BUT INCLUDING ' b '. THIS CORRESPONDS TO INEQUALITIES LIKE $a < x \leq b$.
- $[a, b)$: REPRESENTS ALL NUMBERS BETWEEN ' a ' AND ' b ', INCLUDING ' a ' BUT EXCLUDING ' b '. THIS CORRESPONDS TO INEQUALITIES LIKE $a \leq x < b$.
- (a, ∞) : REPRESENTS ALL NUMBERS GREATER THAN ' a ', EXCLUDING ' a '. THIS CORRESPONDS TO INEQUALITIES LIKE $x > a$.
- $[a, \infty)$: REPRESENTS ALL NUMBERS GREATER THAN OR EQUAL TO ' a ', INCLUDING ' a '. THIS CORRESPONDS TO

INEQUALITIES LIKE $x \geq a$.

- $(-\infty, b)$: REPRESENTS ALL NUMBERS LESS THAN 'b', EXCLUDING 'b'. THIS CORRESPONDS TO INEQUALITIES LIKE $x < b$.
- $(-\infty, b]$: REPRESENTS ALL NUMBERS LESS THAN OR EQUAL TO 'b', INCLUDING 'b'. THIS CORRESPONDS TO INEQUALITIES LIKE $x \leq b$.
- $(-\infty, \infty)$: REPRESENTS ALL REAL NUMBERS. THIS IS USED WHEN THERE ARE NO RESTRICTIONS ON THE VARIABLE.

CONNECTING INEQUALITIES AND INTERVAL NOTATION

THE REAL POWER OF INTERVAL NOTATION BECOMES APPARENT WHEN WE USE IT TO EXPRESS THE SOLUTIONS TO INEQUALITIES. IT PROVIDES A VISUAL AND CONCISE WAY TO COMMUNICATE A RANGE OF NUMBERS THAT SATISFY A GIVEN MATHEMATICAL CONDITION. FOR EXAMPLE, IF WE SOLVE THE INEQUALITY ' $x > 5$ ', THE SOLUTION IS ALL NUMBERS GREATER THAN 5. IN INTERVAL NOTATION, THIS IS WRITTEN AS $(5, \infty)$. THIS IMMEDIATELY TELLS US THAT 5 IS NOT INCLUDED (DUE TO THE PARENTHESIS) AND THAT THE RANGE EXTENDS INFINITELY IN THE POSITIVE DIRECTION.

THIS CONNECTION IS NOT MERELY FOR CONVENIENCE; IT'S FUNDAMENTAL TO HOW WE COMMUNICATE MATHEMATICAL RESULTS. WHEN A PROBLEM ASKS FOR THE SOLUTION SET OF AN INEQUALITY, THE EXPECTED ANSWER IS OFTEN IN INTERVAL NOTATION. IT'S THE STANDARD LANGUAGE FOR DESCRIBING THESE SETS OF NUMBERS, ENSURING THAT EVERYONE READING YOUR WORK UNDERSTANDS PRECISELY WHICH VALUES ARE INCLUDED IN THE SOLUTION AND WHICH ARE NOT.

TRANSLATING INEQUALITIES TO INTERVALS

THE PROCESS OF TRANSLATING AN INEQUALITY INTO INTERVAL NOTATION INVOLVES A FEW STRAIGHTFORWARD STEPS. FIRST, YOU MUST SOLVE THE INEQUALITY TO ISOLATE THE VARIABLE. ONCE YOU HAVE THE VARIABLE ON ONE SIDE AND A NUMBER OR EXPRESSION ON THE OTHER, YOU CAN DETERMINE THE INTERVAL. THE INEQUALITY SYMBOL ITSELF DIRECTLY DICTATES WHETHER YOU'LL USE A PARENTHESIS OR A BRACKET.

FOR INSTANCE, CONSIDER THE INEQUALITY ' $3x - 6 \leq 9$ '. TO SOLVE IT, WE ADD 6 TO BOTH SIDES, YIELDING ' $3x \leq 15$ '. THEN, WE DIVIDE BY 3, RESULTING IN ' $x \leq 5$ '. SINCE THE INEQUALITY SYMBOL IS ' $\leq$ ', WE KNOW THAT 5 IS INCLUDED IN THE SOLUTION. THEREFORE, THE INTERVAL NOTATION IS $(-\infty, 5]$. THE PARENTHESIS ON THE LEFT SIGNIFIES THE UNBOUNDED NATURE TOWARDS NEGATIVE INFINITY, AND THE BRACKET ON THE RIGHT INDICATES THAT 5 IS PART OF THE SOLUTION SET.

REPRESENTING DISJOINT INTERVALS

SOMETIMES, THE SOLUTION TO AN INEQUALITY IS NOT A SINGLE CONTINUOUS RANGE OF NUMBERS BUT RATHER TWO OR MORE SEPARATE INTERVALS. THIS OFTEN OCCURS WITH ABSOLUTE VALUE INEQUALITIES OR QUADRATIC INEQUALITIES. IN SUCH CASES, WE USE THE UNION SYMBOL, DENOTED BY ' $\cup$ ', TO CONNECT THESE DISTINCT INTERVALS. THE UNION SYMBOL ESSENTIALLY MEANS "OR," INDICATING THAT ANY NUMBER IN EITHER OF THE SPECIFIED INTERVALS IS PART OF THE SOLUTION SET.

FOR EXAMPLE, IF WE SOLVED AN INEQUALITY AND FOUND THAT THE SOLUTION WAS ' $x < -2$ ' OR ' $x > 4$ ', WE WOULD REPRESENT THIS IN INTERVAL NOTATION AS $(-\infty, -2) \cup (4, \infty)$. THIS CLEARLY SHOWS TWO SEPARATE SETS OF NUMBERS THAT SATISFY THE INEQUALITY: ALL NUMBERS LESS THAN -2, AND ALL NUMBERS GREATER THAN 4. THE UNION SYMBOL IS CRUCIAL FOR ACCURATELY CONVEYING THESE TYPES OF COMPOUND SOLUTIONS.

SOLVING LINEAR INEQUALITIES USING INTERVAL NOTATION

SOLVING LINEAR INEQUALITIES AND EXPRESSING THEIR SOLUTIONS USING INTERVAL NOTATION IS A CORE SKILL IN COLLEGE ALGEBRA. THE PROCESS IS VERY SIMILAR TO SOLVING LINEAR EQUATIONS, WITH ONE CRITICAL DIFFERENCE: YOU MUST PAY CLOSE ATTENTION TO THE DIRECTION OF THE INEQUALITY SIGN. WHEN YOU MULTIPLY OR DIVIDE BOTH SIDES OF AN INEQUALITY BY A NEGATIVE NUMBER, YOU MUST REVERSE THE INEQUALITY SYMBOL. THIS RULE ENSURES THAT THE RELATIONSHIP BETWEEN THE TWO SIDES REMAINS ACCURATE.

ONCE THE INEQUALITY IS SOLVED AND THE VARIABLE IS ISOLATED, THE RESULTING INEQUALITY CAN BE DIRECTLY TRANSLATED INTO INTERVAL NOTATION. THIS PROVIDES A CLEAR, CONCISE, AND UNIVERSALLY UNDERSTOOD WAY TO REPRESENT THE INFINITE SET OF NUMBERS THAT SATISFY THE INEQUALITY. MASTERY OF THIS PROCESS IS FOUNDATIONAL FOR UNDERSTANDING MORE COMPLEX INEQUALITY TYPES AND THEIR GRAPHICAL REPRESENTATIONS.

STEP-BY-STEP SOLUTION PROCESS

LET'S WALK THROUGH AN EXAMPLE TO SOLIDIFY THE PROCESS. SUPPOSE WE NEED TO SOLVE THE INEQUALITY ' $5 - 2x < 11$ ' AND EXPRESS THE SOLUTION IN INTERVAL NOTATION.

1. **ISOLATE THE VARIABLE TERM:** SUBTRACT 5 FROM BOTH SIDES OF THE INEQUALITY:

$$-2x < 11 - 5$$

$$-2x < 6$$

2. **ISOLATE THE VARIABLE:** DIVIDE BOTH SIDES BY -2. REMEMBER TO REVERSE THE INEQUALITY SIGN BECAUSE WE ARE DIVIDING BY A NEGATIVE NUMBER:

$$x > 6 / -2$$

$$x > -3$$

3. **TRANSLATE TO INTERVAL NOTATION:** THE INEQUALITY ' $x > -3$ ' MEANS ALL NUMBERS GREATER THAN -3. SINCE -3 IS NOT INCLUDED (DUE TO THE ' $>$ ' SIGN), WE USE A PARENTHESIS. THE INTERVAL EXTENDS INFINITELY TO THE RIGHT. THE INTERVAL NOTATION IS $(-3, \infty)$.

THIS STEP-BY-STEP APPROACH ENSURES ACCURACY AND HELPS BUILD CONFIDENCE IN SOLVING LINEAR INEQUALITIES AND EXPRESSING THEIR SOLUTIONS CORRECTLY.

COMMON PITFALLS IN LINEAR INEQUALITIES

ONE OF THE MOST COMMON MISTAKES STUDENTS MAKE WHEN SOLVING LINEAR INEQUALITIES IS FORGETTING TO REVERSE THE INEQUALITY SIGN WHEN MULTIPLYING OR DIVIDING BY A NEGATIVE NUMBER. THIS CAN LEAD TO AN ENTIRELY INCORRECT SOLUTION SET. ALWAYS DOUBLE-CHECK THIS STEP, ESPECIALLY IF YOU FIND YOURSELF DIVIDING OR MULTIPLYING BY A NEGATIVE VALUE.

ANOTHER POTENTIAL PITFALL IS INCORRECTLY TRANSLATING THE FINAL INEQUALITY INTO INTERVAL NOTATION. FORGETTING WHETHER TO USE A PARENTHESIS OR A BRACKET, OR MISINTERPRETING THE DIRECTION OF INFINITY, CAN RENDER THE ANSWER

INCORRECT EVEN IF THE ALGEBRAIC MANIPULATION WAS PERFECT. CAREFULLY REVIEW THE INEQUALITY SYMBOL ($>$, $<$, $\geq$, $\leq$) AND MATCH IT WITH THE CORRESPONDING NOTATION (PARENTHESES OR BRACKETS) AND DIRECTION OF INFINITY.

SOLVING COMPOUND INEQUALITIES WITH INTERVAL NOTATION

COMPOUND INEQUALITIES ARE FORMED BY JOINING TWO OR MORE INEQUALITIES WITH CONJUNCTIONS LIKE "AND" OR "OR." WHEN SOLVING THESE, WE OFTEN NEED TO FIND THE INTERSECTION (FOR "AND") OR THE UNION (FOR "OR") OF THE SOLUTION SETS OF THE INDIVIDUAL INEQUALITIES. INTERVAL NOTATION IS EXCEPTIONALLY WELL-SUITED FOR REPRESENTING THESE MORE COMPLEX SOLUTION SETS, ESPECIALLY WHEN THEY INVOLVE MULTIPLE, DISTINCT INTERVALS.

UNDERSTANDING HOW TO COMBINE THESE INDIVIDUAL SOLUTION SETS IS KEY. THE "AND" CONJUNCTION REQUIRES VALUES TO SATISFY BOTH CONDITIONS SIMULTANEOUSLY, LEADING TO AN INTERSECTION OF INTERVALS. THE "OR" CONJUNCTION, ON THE OTHER HAND, MEANS A VALUE NEEDS TO SATISFY AT LEAST ONE OF THE CONDITIONS, LEADING TO A UNION OF INTERVALS. MASTERING THESE DISTINCTIONS AND APPLYING INTERVAL NOTATION WILL ALLOW YOU TO ACCURATELY DESCRIBE THE SOLUTIONS TO A BROAD RANGE OF PROBLEMS.

COMPOUND INEQUALITIES WITH "AND"

COMPOUND INEQUALITIES CONNECTED BY "AND" ARE OFTEN WRITTEN IN A COMPACT FORM, LIKE ' $a < x < b$ ', OR AS TWO SEPARATE INEQUALITIES THAT MUST BOTH BE TRUE. THE SOLUTION SET IS THE INTERSECTION OF THE SOLUTION SETS OF THE INDIVIDUAL INEQUALITIES. THIS MEANS WE ARE LOOKING FOR THE NUMBERS THAT ARE COMMON TO BOTH RANGES.

FOR EXAMPLE, CONSIDER THE COMPOUND INEQUALITY ' $x > 2$ AND $x < 7$ '. THE SOLUTION FOR ' $x > 2$ ' IS $(2, \infty)$, AND THE SOLUTION FOR ' $x < 7$ ' IS $(-\infty, 7)$. THE INTERSECTION OF THESE TWO SETS IS $(2, 7)$. THIS IS BECAUSE ONLY THE NUMBERS THAT ARE BOTH GREATER THAN 2 AND LESS THAN 7 FALL WITHIN THIS COMBINED RANGE. IF THE COMPOUND INEQUALITY WAS WRITTEN AS ' $2 < x < 7$ ', THE INTERVAL NOTATION $(2, 7)$ DIRECTLY REPRESENTS THIS INTERSECTION.

COMPOUND INEQUALITIES WITH "OR"

COMPOUND INEQUALITIES CONNECTED BY "OR" REQUIRE THAT AT LEAST ONE OF THE INEQUALITIES BE TRUE. THE SOLUTION SET IS THE UNION OF THE SOLUTION SETS OF THE INDIVIDUAL INEQUALITIES. THIS MEANS ANY NUMBER THAT SATISFIES EITHER ONE OF THE CONDITIONS, OR BOTH, IS PART OF THE OVERALL SOLUTION.

CONSIDER THE COMPOUND INEQUALITY ' $x \leq -1$ OR $x \geq 3$ '. THE SOLUTION FOR ' $x \leq -1$ ' IS $(-\infty, -1]$, AND THE SOLUTION FOR ' $x \geq 3$ ' IS $[3, \infty)$. SINCE THESE INTERVALS DO NOT OVERLAP, THE UNION IS SIMPLY WRITTEN BY COMBINING THEM WITH THE UNION SYMBOL: $(-\infty, -1] \cup [3, \infty)$. THIS NOTATION CLEARLY INDICATES THAT THE SOLUTION CONSISTS OF ALL NUMBERS LESS THAN OR EQUAL TO -1, ALONG WITH ALL NUMBERS GREATER THAN OR EQUAL TO 3.

ABSOLUTE VALUE INEQUALITIES AND INTERVAL NOTATION

ABSOLUTE VALUE INEQUALITIES ADD ANOTHER LAYER OF COMPLEXITY, BUT INTERVAL NOTATION PROVIDES A CLEAR WAY TO EXPRESS THEIR SOLUTIONS. REMEMBER, THE ABSOLUTE VALUE OF A NUMBER REPRESENTS ITS DISTANCE FROM ZERO. SO, AN INEQUALITY LIKE $|x - a| < b$ MEANS THAT THE DISTANCE BETWEEN ' x ' AND ' a ' IS LESS THAN ' b '. THIS TYPICALLY RESULTS IN A SINGLE, BOUNDED INTERVAL.

CONVERSELY, AN INEQUALITY LIKE $|x - a| > b$ MEANS THAT THE DISTANCE BETWEEN ' x ' AND ' a ' IS GREATER THAN ' b '. THIS USUALLY LEADS TO TWO SEPARATE INTERVALS, ONE ON EACH "SIDE" OF A CERTAIN RANGE. WE'LL EXPLORE HOW TO

TRANSLATE THESE INTO PRECISE INTERVAL NOTATION.

SOLVING $|AX + B| < C$ AND $|AX + B| \leq C$

INEQUALITIES OF THE FORM $|AX + B| < C$ OR $|AX + B| \leq C$ ARE SOLVED BY CONVERTING THEM INTO A COMPOUND INEQUALITY WITH "AND." SPECIFICALLY, $|AX + B| < C$ IS EQUIVALENT TO $-C < AX + B < C$. THE SOLUTION SET WILL BE A SINGLE, BOUNDED INTERVAL.

LET'S SOLVE $|2x - 1| < 5$. THIS IS EQUIVALENT TO $-5 < 2x - 1 < 5$.

1. ADD 1 TO ALL PARTS: $-5 + 1 < 2x < 5 + 1$, WHICH SIMPLIFIES TO $-4 < 2x < 6$.
2. DIVIDE ALL PARTS BY 2: $-4/2 < x < 6/2$, WHICH GIVES $-2 < x < 3$.

THE INTERVAL NOTATION FOR THIS SOLUTION IS $(-2, 3)$. IF THE ORIGINAL INEQUALITY HAD BEEN $|2x - 1| \leq 5$, THE INTERVAL WOULD HAVE BEEN $[-2, 3]$.

SOLVING $|AX + B| > C$ AND $|AX + B| \geq C$

INEQUALITIES OF THE FORM $|AX + B| > C$ OR $|AX + B| \geq C$ ARE SOLVED BY CONVERTING THEM INTO TWO SEPARATE INEQUALITIES CONNECTED BY "OR." SPECIFICALLY, $|AX + B| > C$ IS EQUIVALENT TO $AX + B > C$ OR $AX + B < -C$. THE SOLUTION SET WILL TYPICALLY CONSIST OF TWO DISJOINT INTERVALS.

LET'S SOLVE $|x + 3| > 2$. THIS IS EQUIVALENT TO $x + 3 > 2$ OR $x + 3 < -2$.

1. SOLVE THE FIRST INEQUALITY: $x + 3 > 2 \Rightarrow x > -1$.
2. SOLVE THE SECOND INEQUALITY: $x + 3 < -2 \Rightarrow x < -5$.

THE SOLUTION IS ' $x > -1$ ' OR ' $x < -5$ '. IN INTERVAL NOTATION, THIS IS REPRESENTED AS THE UNION OF TWO INTERVALS: $(-5, -1)$. IF THE ORIGINAL INEQUALITY HAD BEEN $|x + 3| \geq 2$, THE SOLUTION WOULD HAVE BEEN $(-\infty, -5] \cup [-1, \infty)$.

QUADRATIC INEQUALITIES AND INTERVAL NOTATION

QUADRATIC INEQUALITIES, WHICH INVOLVE A TERM WITH THE VARIABLE SQUARED (LIKE x^2), REQUIRE A SYSTEMATIC APPROACH TO FIND THEIR SOLUTION SETS. THE KEY TO SOLVING THEM LIES IN IDENTIFYING THE "CRITICAL POINTS" OR ROOTS OF THE RELATED QUADRATIC EQUATION. THESE ROOTS DIVIDE THE NUMBER LINE INTO INTERVALS, AND WE THEN TEST EACH INTERVAL TO DETERMINE WHERE THE INEQUALITY HOLDS TRUE.

INTERVAL NOTATION IS INDISPENSABLE HERE, AS THE SOLUTIONS OFTEN COMPRISE ONE OR TWO DISTINCT INTERVALS. ACCURATELY REPRESENTING THESE INTERVALS AND THEIR UNIONS IS CRUCIAL FOR A COMPLETE AND CORRECT ANSWER. THE PROCESS MIGHT SEEM A BIT MORE INVOLVED THAN WITH LINEAR INEQUALITIES, BUT ONCE YOU GRASP THE METHOD, IT BECOMES A POWERFUL TOOL FOR ANALYZING QUADRATIC RELATIONSHIPS.

FINDING CRITICAL POINTS

THE FIRST STEP IN SOLVING A QUADRATIC INEQUALITY IS TO FIND THE ROOTS OF THE CORRESPONDING QUADRATIC EQUATION. THESE ROOTS ARE THE VALUES OF THE VARIABLE THAT MAKE THE EXPRESSION EQUAL TO ZERO. FOR EXAMPLE, TO SOLVE $x^2 - 5x + 6 > 0$, WE FIRST CONSIDER THE EQUATION $x^2 - 5x + 6 = 0$. FACTORING THIS EQUATION, WE GET $(x - 2)(x - 3) = 0$. THE ROOTS ARE $x = 2$ AND $x = 3$. THESE ROOTS ARE OUR CRITICAL POINTS.

THESE CRITICAL POINTS ARE SIGNIFICANT BECAUSE THEY ARE THE ONLY POINTS WHERE THE QUADRATIC EXPRESSION CAN CHANGE ITS SIGN (FROM POSITIVE TO NEGATIVE, OR VICE VERSA). THEREFORE, THEY DIVIDE THE NUMBER LINE INTO DISTINCT INTERVALS WHERE THE EXPRESSION WILL CONSISTENTLY HAVE THE SAME SIGN. UNDERSTANDING THIS HELPS US SYSTEMATICALLY CHECK FOR SOLUTIONS.

TESTING INTERVALS

ONCE YOU HAVE IDENTIFIED THE CRITICAL POINTS, YOU DIVIDE THE NUMBER LINE INTO INTERVALS BASED ON THESE POINTS. FOR OUR EXAMPLE $x^2 - 5x + 6 > 0$, THE CRITICAL POINTS 2 AND 3 DIVIDE THE NUMBER LINE INTO THREE INTERVALS: $(-\infty, 2)$, $(2, 3)$, AND $(3, \infty)$. THE NEXT STEP IS TO CHOOSE A TEST VALUE FROM EACH INTERVAL AND SUBSTITUTE IT BACK INTO THE ORIGINAL INEQUALITY TO SEE IF IT MAKES THE STATEMENT TRUE.

LET'S TEST VALUES:

- FOR $(-\infty, 2)$, LET'S TEST $x = 0$: $(0)^2 - 5(0) + 6 = 6$. IS $6 > 0$? YES. SO, THIS INTERVAL IS PART OF THE SOLUTION.
- FOR $(2, 3)$, LET'S TEST $x = 2.5$: $(2.5)^2 - 5(2.5) + 6 = 6.25 - 12.5 + 6 = -0.25$. IS $-0.25 > 0$? NO. SO, THIS INTERVAL IS NOT PART OF THE SOLUTION.
- FOR $(3, \infty)$, LET'S TEST $x = 4$: $(4)^2 - 5(4) + 6 = 16 - 20 + 6 = 2$. IS $2 > 0$? YES. SO, THIS INTERVAL IS PART OF THE SOLUTION.

THEREFORE, THE SOLUTION TO $x^2 - 5x + 6 > 0$ IS THE UNION OF THE INTERVALS THAT TESTED TRUE: $(-\infty, 2) \cup (3, \infty)$. IF THE INEQUALITY HAD BEEN $\geq$, WE WOULD INCLUDE THE ENDPOINTS 2 AND 3 IF THEY MADE THE ORIGINAL INEQUALITY TRUE (WHICH THEY DO, AS THEY RESULT IN 0, AND $0 \geq 0$ IS TRUE).

GRAPHING INEQUALITIES ON A NUMBER LINE

WHILE INTERVAL NOTATION PROVIDES A SYMBOLIC REPRESENTATION OF INEQUALITY SOLUTIONS, GRAPHING THEM ON A NUMBER LINE OFFERS A VISUAL CONFIRMATION. THIS VISUAL REPRESENTATION IS INCREDIBLY HELPFUL FOR UNDERSTANDING THE RANGE OF VALUES AND HOW THEY RELATE TO EACH OTHER. IT'S LIKE DRAWING A MAP OF YOUR SOLUTION SET.

THE GRAPHING PROCESS USES OPEN CIRCLES AND CLOSED CIRCLES, ALONG WITH SHADED LINES, TO DEPICT THE INTERVALS. OPEN CIRCLES CORRESPOND TO PARENTHESES IN INTERVAL NOTATION (EXCLUSIVE ENDPOINTS), WHILE CLOSED CIRCLES CORRESPOND TO BRACKETS (INCLUSIVE ENDPOINTS). THE SHADING INDICATES THE DIRECTION OF THE SOLUTION SET.

NUMBER LINE CONVENTIONS

WHEN GRAPHING AN INEQUALITY ON A NUMBER LINE, YOU'LL PRIMARILY USE TWO SYMBOLS: AN OPEN CIRCLE AND A CLOSED CIRCLE. AN OPEN CIRCLE IS PLACED AT A SPECIFIC NUMBER AND INDICATES THAT THIS NUMBER IS NOT INCLUDED IN THE SOLUTION SET. THIS CORRESPONDS TO STRICT INEQUALITIES LIKE ' $<$ ' AND ' $>$ '. THINK OF IT AS AN EMPTY SPACE AT THAT POINT.

A CLOSED CIRCLE, ON THE OTHER HAND, IS PLACED AT A SPECIFIC NUMBER AND SIGNIFIES THAT THIS NUMBER IS INCLUDED IN THE SOLUTION SET. THIS IS USED FOR INEQUALITIES THAT INCLUDE "OR EQUAL TO," SUCH AS ' $\leq$ ' AND ' $\geq$ '. IT'S LIKE FILLING IN THAT POINT, MAKING IT PART OF THE VALID SOLUTIONS.

VISUALIZING INTERVAL NOTATION ON THE LINE

LET'S CONNECT THE GRAPHING WITH INTERVAL NOTATION. IF YOU HAVE THE INTERVAL (A, B) , YOU WOULD DRAW A NUMBER LINE, PLACE AN OPEN CIRCLE AT 'A' AND ANOTHER OPEN CIRCLE AT 'B', AND THEN SHADE THE REGION BETWEEN THEM. THIS VISUALLY REPRESENTS ALL NUMBERS STRICTLY BETWEEN 'A' AND 'B'.

FOR THE INTERVAL $[A, B]$, YOU WOULD PLACE A CLOSED CIRCLE AT 'A' AND A CLOSED CIRCLE AT 'B', SHADING THE REGION BETWEEN THEM. THIS SHOWS THAT BOTH 'A' AND 'B' ARE INCLUDED IN THE SOLUTION.

FOR UNBOUNDED INTERVALS LIKE (A, ∞) , YOU WOULD PLACE AN OPEN CIRCLE AT 'A' AND DRAW AN ARROW EXTENDING INFINITELY TO THE RIGHT, INDICATING ALL NUMBERS GREATER THAN 'A'. FOR $[-\infty, B]$, YOU WOULD PLACE A CLOSED CIRCLE AT 'B' AND DRAW AN ARROW EXTENDING INFINITELY TO THE LEFT, REPRESENTING ALL NUMBERS LESS THAN OR EQUAL TO 'B'. THE COMBINATION OF OPEN/CLOSED CIRCLES AND SHADING PROVIDES A CLEAR, INTUITIVE PICTURE OF THE SOLUTION SET.

PRACTICAL APPLICATIONS OF INTERVAL NOTATION AND INEQUALITIES

THE CONCEPTS OF INEQUALITIES AND INTERVAL NOTATION ARE NOT JUST ABSTRACT MATHEMATICAL EXERCISES; THEY HAVE NUMEROUS REAL-WORLD APPLICATIONS. IN FIELDS RANGING FROM PHYSICS AND ENGINEERING TO ECONOMICS AND COMPUTER SCIENCE, UNDERSTANDING AND APPLYING THESE PRINCIPLES IS CRUCIAL FOR PROBLEM-SOLVING AND DECISION-MAKING.

THINK ABOUT SITUATIONS WHERE YOU DEAL WITH LIMITS, TOLERANCES, OR RANGES OF ACCEPTABLE VALUES. INEQUALITIES PROVIDE THE MATHEMATICAL LANGUAGE TO DESCRIBE THESE SCENARIOS, AND INTERVAL NOTATION OFFERS A CLEAR WAY TO COMMUNICATE THE SOLUTIONS. WHETHER IT'S DETERMINING THE POSSIBLE OPERATING TEMPERATURES FOR A DEVICE OR ANALYZING THE RANGE OF PRICES FOR A STOCK, THESE TOOLS ARE INVALUABLE.

IN SCIENCE AND ENGINEERING

IN SCIENCE AND ENGINEERING, INEQUALITIES ARE USED EXTENSIVELY TO DEFINE OPERATIONAL PARAMETERS, SAFETY LIMITS, AND PHYSICAL CONSTRAINTS. FOR EXAMPLE, A CIVIL ENGINEER MIGHT SPECIFY THAT THE LOAD-BEARING CAPACITY OF A BRIDGE MUST BE GREATER THAN OR EQUAL TO A CERTAIN WEIGHT (E.G., $\text{Load} \geq 100 \text{ tons}$), OR THAT THE TEMPERATURE WITHIN A SENSITIVE PIECE OF EQUIPMENT MUST REMAIN WITHIN A SPECIFIC RANGE (E.G., $20^\circ\text{C} \leq \text{Temperature} \leq 25^\circ\text{C}$). INTERVAL NOTATION IS THEN USED TO CONCISELY REPRESENT THESE REQUIRED RANGES, MAKING THEM EASY TO UNDERSTAND AND IMPLEMENT IN DESIGNS AND CALCULATIONS.

SIMILARLY, IN PHYSICS, CONCEPTS LIKE VELOCITY LIMITS, ENERGY REQUIREMENTS, OR THE RANGE OF MOTION FOR A ROBOTIC ARM CAN ALL BE EXPRESSED USING INEQUALITIES. THE SOLUTIONS TO THESE INEQUALITIES, OFTEN REPRESENTED BY INTERVAL NOTATION, ARE CRITICAL FOR DESIGNING SAFE AND EFFECTIVE SYSTEMS.

IN ECONOMICS AND FINANCE

THE WORLD OF ECONOMICS AND FINANCE IS RIFE WITH SCENARIOS THAT CAN BE MODELED USING INEQUALITIES. FOR INSTANCE, A COMPANY MIGHT SET A PROFIT TARGET THAT MUST BE MET OR EXCEEDED ($\text{Profit} \geq \$1 \text{ million}$). A FINANCIAL ADVISOR MIGHT RECOMMEND INVESTMENT PORTFOLIOS THAT FALL WITHIN A CERTAIN RISK TOLERANCE ($\text{Risk Level} \leq \text{Moderate}$). CONSUMER

DEMAND MIGHT BE MODELED AS A FUNCTION OF PRICE, WITH CERTAIN PRICE POINTS LEADING TO A MINIMUM LEVEL OF SALES (PRICE < \$50 IMPLIES SALES $\geq$ 1000 UNITS).

INTERVAL NOTATION HELPS TO CLEARLY DEFINE THESE ACCEPTABLE RANGES FOR FINANCIAL PERFORMANCE, RISK LEVELS, OR MARKET CONDITIONS. THIS ALLOWS FOR BETTER STRATEGIC PLANNING, RISK MANAGEMENT, AND INFORMED DECISION-MAKING IN THE FINANCIAL SECTOR.

IN EVERYDAY LIFE

EVEN IN OUR DAILY LIVES, WE ENCOUNTER SITUATIONS WHERE INEQUALITIES AND INTERVAL NOTATION ARE IMPLICITLY USED. FOR EXAMPLE, WHEN A SIGN SAYS "CHILDREN UNDER 12 PAY HALF PRICE," IT IMPLIES AN INEQUALITY FOR AGE. IF THE PRICE IS \$10, THEN FOR AGES $0 \leq \text{AGE} < 12$, THE COST IS \$5. FOR ALL OTHER AGES ($\text{AGE} \geq 12$), THE COST IS \$10.

SPEED LIMITS ARE ANOTHER PRIME EXAMPLE: 65 MPH MEANS YOUR SPEED MUST BE LESS THAN OR EQUAL TO 65 ($\text{SPEED} \leq 65$). IF THERE'S ALSO A MINIMUM SPEED ON A HIGHWAY, SAY 40 MPH, THEN YOUR SPEED MUST BE WITHIN THE INTERVAL $[40, 65]$. THESE EVERYDAY EXAMPLES DEMONSTRATE HOW THESE MATHEMATICAL CONCEPTS ARE WOVEN INTO THE FABRIC OF OUR UNDERSTANDING OF LIMITS AND RANGES.

FAQ

Q: WHAT IS THE PRIMARY DIFFERENCE BETWEEN A STRICT INEQUALITY AND A NON-STRICT INEQUALITY?

A: A STRICT INEQUALITY USES SYMBOLS LIKE $<$ (LESS THAN) OR $>$ (GREATER THAN), MEANING THE VALUES ON EITHER SIDE CANNOT BE EQUAL. A NON-STRICT INEQUALITY USES SYMBOLS LIKE $\leq$ (LESS THAN OR EQUAL TO) OR $\geq$ (GREATER THAN OR EQUAL TO), MEANING THE VALUES ON EITHER SIDE CAN BE EQUAL. THIS DISTINCTION IS CRUCIAL FOR DETERMINING WHETHER TO USE PARENTHESES OR BRACKETS IN INTERVAL NOTATION.

Q: WHY IS INTERVAL NOTATION PREFERRED OVER JUST WRITING THE INEQUALITY FOR SOLUTIONS?

A: INTERVAL NOTATION IS PREFERRED BECAUSE IT'S A MORE CONCISE AND STANDARDIZED WAY TO REPRESENT SETS OF NUMBERS, ESPECIALLY INFINITE SETS. IT CLEARLY INDICATES THE ENDPOINTS AND WHETHER THEY ARE INCLUDED, MAKING IT UNAMBIGUOUS AND UNIVERSALLY UNDERSTOOD IN MATHEMATICS. IT ALSO SIMPLIFIES THE VISUAL REPRESENTATION ON A NUMBER LINE.

Q: CAN THE SOLUTION TO A SINGLE LINEAR INEQUALITY RESULT IN TWO SEPARATE INTERVALS?

A: NO, THE SOLUTION TO A SINGLE LINEAR INEQUALITY WILL ALWAYS BE A SINGLE INTERVAL (OR ALL REAL NUMBERS). TWO SEPARATE INTERVALS TYPICALLY ARISE FROM ABSOLUTE VALUE INEQUALITIES OR QUADRATIC INEQUALITIES THAT ARE SPLIT INTO "OR" CONDITIONS.

Q: WHAT DOES THE UNION SYMBOL ($\cup$) MEAN WHEN USED IN INTERVAL NOTATION?

A: THE UNION SYMBOL ($\cup$) SIGNIFIES "OR" IN INTERVAL NOTATION. IT'S USED TO COMBINE TWO OR MORE DISJOINT INTERVALS, MEANING THAT ANY NUMBER FALLING INTO ANY OF THOSE SPECIFIED INTERVALS IS CONSIDERED PART OF THE OVERALL SOLUTION SET.

Q: HOW DO I REPRESENT AN INEQUALITY LIKE $x \geq 5$ USING INTERVAL NOTATION?

A: THE INEQUALITY $x \geq 5$ MEANS ALL NUMBERS GREATER THAN OR EQUAL TO 5. IN INTERVAL NOTATION, THIS IS WRITTEN AS $[5, \infty)$. THE BRACKET AT 5 INDICATES THAT 5 IS INCLUDED, AND THE INFINITY SYMBOL WITH A PARENTHESIS INDICATES THAT THE INTERVAL EXTENDS INFINITELY TO THE RIGHT WITHOUT AN UPPER BOUND.

Q: WHEN SOLVING ABSOLUTE VALUE INEQUALITIES, HOW DO I KNOW WHETHER TO USE "AND" OR "OR"?

A: FOR ABSOLUTE VALUE INEQUALITIES INVOLVING ' $<$ ' OR ' $\leq$ ' (E.G., $|expression| < c$), YOU USE "AND," WHICH LEADS TO A COMPOUND INEQUALITY LIKE $-c < expression < c$, TYPICALLY RESULTING IN A SINGLE, BOUNDED INTERVAL. FOR ABSOLUTE VALUE INEQUALITIES INVOLVING ' $>$ ' OR ' $\geq$ ' (E.G., $|expression| > c$), YOU USE "OR," WHICH LEADS TO TWO SEPARATE INEQUALITIES ($expression > c$ OR $expression < -c$), TYPICALLY RESULTING IN TWO DISJOINT INTERVALS.

Q: WHAT IS THE SIGNIFICANCE OF THE CRITICAL POINTS IN SOLVING QUADRATIC INEQUALITIES?

A: CRITICAL POINTS ARE THE ROOTS OF THE RELATED QUADRATIC EQUATION. THEY ARE SIGNIFICANT BECAUSE THEY ARE THE ONLY POINTS WHERE THE QUADRATIC EXPRESSION CAN CHANGE ITS SIGN. THESE POINTS DIVIDE THE NUMBER LINE INTO INTERVALS, AND WITHIN EACH INTERVAL, THE SIGN OF THE QUADRATIC EXPRESSION REMAINS CONSTANT, ALLOWING US TO EFFICIENTLY TEST FOR SOLUTIONS.

Q: IF AN INEQUALITY INVOLVES "LESS THAN OR EQUAL TO" ($\leq$) OR "GREATER THAN OR EQUAL TO" ($\geq$), WHAT SYMBOL DO I USE IN INTERVAL NOTATION?

A: FOR INEQUALITIES INVOLVING "LESS THAN OR EQUAL TO" ($\leq$) OR "GREATER THAN OR EQUAL TO" ($\geq$), YOU USE A BRACKET ($[$ OR $]$) IN INTERVAL NOTATION TO INDICATE THAT THE ENDPOINT IS INCLUDED IN THE SOLUTION SET. FOR STRICT INEQUALITIES ($<$ OR $>$), YOU USE A PARENTHESIS.

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